0.1 `ei.RxC`: Hierarchical Multinomial-Dirichlet Ecological Inference Model for $R \times C$ Tables

Given n contingency tables, each with observed marginals (column and row totals), ecological inference (EI) estimates the internal cell values in each table. The hierarchical Multinomial-Dirichlet model estimates cell counts in $R \times C$ tables. The model is implemented using a nonlinear least squares approximation and, with bootstrapping for standard errors, had good frequentist properties.

Syntax

```
> z.out <- zelig(cbind(T0, T1, T2, T3) ~ X0 + X1,
                 covar = NULL,
                 model = "eiRxC", data = mydata)
> x.out <- setx(z.out, fn = NULL)
> s.out <- sim(z.out)
```

Inputs

- `T0`, `T1`, `T2`,..., `TC`: numeric vectors (either counts, or proportions that sum to one for each row) containing the column margins of the units to be analyzed.
- `X0`, `X1`, `X2`,...,`XR`: numeric vectors (either counts, or proportions that sum to one for each row) containing the row margins of the units to be analyzed.
- `covar`: (optional) a covariate that varies across tables, specified as `covar = ~ Z1`, for example. (The model only accepts one covariate.)

Examples

1. Basic examples: No covariate
   Attaching the example dataset:

   ```
   > data(Weimar)
   ```

   Estimating the model:

   ```
   > z.out <- zelig(cbind(Nazi, Government, Communists, FarRight,
   +     Other) ~ shareunemployed + shareblue + sharewhite + shareself +
   +     sharedomestic, model = "ei.RxC", data = Weimar)
   > summary(z.out)
   ```

   Setting values for in-sample simulations given marginal values:

   ```
   > x.out <- setx(z.out)
   ```

Estimate fractions of different social groups that support political parties:

```
> s.out <- sim(z.out)
```

Summarizing fractions of different social groups that support political parties:

```
> summary(s.out)
```

2. Example of covariates being present in the model

   Using the example dataset Weimar and estimating the model

```
> z.out <- zelig(cbind(Nazi, Government, Communists, FarRight,
+     Other) ~ shareunemployed + shareblue + sharewhite + shareself +
+     sharedomestic, covar = ~shareprotestants, model = "ei.RxC",
+     data = Weimar)
> summary(z.out)
```

   Set the covariate to its default (mean/median) value

```
> x.out <- setx(z.out)
```

   Estimate fractions of different social groups that support political parties:

```
> s.out <- sim(z.out)
```

   Summarizing fractions of different social groups that support political parties:

```
> summary(s.out)
```

Model

Consider the following 5×5 contingency table for the voting patterns in Weimar Germany. For each geographical unit i ($i = 1, \ldots, p$), the marginals $T_{1i}, \ldots, T_{Ci}$, $X_{1i}, \ldots, X_{Ri}$ are known for each of the p electoral precincts, and we would like to estimate ($\beta_i^{rc}, r = 1, \ldots, R, c = 1, \ldots, C-1$) which are the fractions of people in social class r who vote for party c, for all r and c.

	Nazi	Government	Communists	Far Right	Other	
Unemployed	β_{11}^i	β_{12}^i	β_{13}^i	β_{14}^i	$1 - \sum_{c=1}^4 \beta_{1c}^i$	X_1^i
Blue	β_{21}^i	β_{22}^i	β_{23}^i	β_{24}^i	$1 - \sum_{c=1}^4 \beta_{2c}^i$	X_2^i
White	β_{31}^i	β_{32}^i	β_{33}^i	β_{34}^i	$1 - \sum_{c=1}^4 \beta_{3c}^i$	X_3^i
Self	β_{41}^i	β_{42}^i	β_{43}^i	β_{44}^i	$1 - \sum_{c=1}^4 \beta_{4c}^i$	X_4^i
Domestic	β_{51}^i	β_{52}^i	β_{53}^i	β_{54}^i	$1 - \sum_{c=1}^4 \beta_{5c}^i$	X_5^i
	T_{1i}	T_{2i}	T_{3i}	T_{4i}	$1 - \sum_{c=1}^4 \beta_{ci}$	

The marginal values $X_{1i}, \ldots, X_{Ri}$, $T_{1i}, \ldots, T_{Ci}$ may be observed as counts or fractions.

Let $T_i^{'} = (T_{1i}^{'}, T_{2i}^{'}, \ldots, T_{Ci}^{'})$ be the number of voting age persons who turn out to vote for different parties. There are three levels of hierarchy in the Multinomial-Dirichlet EI model. At the first stage, we model the data as:

- The *stochastic component* is described $T_i^{'}$ which follows a multinomial distribution:

$$T_i^{'} \sim \text{Multinomial}(\Theta_{1i}, \ldots, \Theta_{Ci})$$

- The *systematic components* are

$$\Theta_{ci} = \sum_{r=1}^{R} \beta_{rc}^i X_{ri} \quad \text{for} \quad c = 1, \ldots, C$$

At the second stage, we use an optional covariate to model Θ_{ci}'s and β_{qrc}^i:

- The *stochastic component* is described by $\beta_r^i = (\beta_{r1}, \beta_{r2}, \ldots, \beta_{r,C-1})$ for $i = 1, \ldots, , p$ and $r = 1, \ldots, R$, which follows a Dirichlet distribution:

$$\beta_r^i \sim \text{Dirichlet}(\alpha_{r1}^i, \ldots, \alpha_{rc}^i)$$

- The *systematic components* are

$$\alpha_{rc}^i = \frac{d_r exp(\gamma_{rc} + \delta_{rc} Z_i)}{d_r(1 + \sum_{j=1}^{C-1} exp(\gamma_{rj} + \delta_{rj} Z_i))} = \frac{exp(\gamma_{rc} + \delta_{rc} Z_i)}{1 + \sum_{j=1}^{C-1} exp(\gamma_{rj} + \delta_{rj} Z_i)}$$

  for $i = 1, \ldots, p$, $r = 1, \ldots, R$, and $c = 1, \ldots, C-1$.

  In the third stage, we assume that the regression parameters (the γ_{rc}'s and δ_{rc}'s) are *a priori* independent, and put a flat prior on these regression parameters. The parameters d_r for $r = 1, \ldots, R$ are assumed to follow exponential distributions with mean $\frac{1}{\lambda}$.

Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run

```
> z.out <- zelig(cbind(T0, T1, T2) ~ X0 + X1 + X2,
          model = "eiRxC", data = mydata)
```

then you may examine the available information in `z.out` by using `names(z.out)`. For example,

- From the `zelig()` output object `z.out$coefficients` are the estimates of γ_{ij} (and also δ_{ij}, if covariates are present). The parameters are returned as a single vector of length $R \times (C - 1)$. If there is a covariate, δ is concatenated to it.
- From the `sim()` output object, you may extract the parameters β_{ij} corresponding to the estimated fractions of different social groups that support different political parties, by using `s.out$qi$ev`. For each precinct, that will be a matrix with dimensions: simulations $\times R \times C$.

Contributors

Please cite the model as

.

Jason Wittenberg, Ferdinand Alimadhi, and Olivia Lau implemented the $R \times C$ EI model for Zelig.